

DOCUMENT No. 4.5

DIFFERENCE TOPOLOGY v1.0 EN

Formal Differentics Mathematical Corpus

Project: Formal Differentics

Author: Giedrius Grebliunas

ORCID: 0009-0002-2135-6420

Year: 2026

Version: v1.0

Language: English

License: CC BY-NC-ND 4.0 International

HASHCODE (SHA-256 Hex):

Abstract

This document introduces Difference Topology, the branch of Formal Differentics concerned with global structural properties of Difference Space that remain invariant under continuous transformations.

While Difference Metrics defines quantitative measures of difference and Difference Geometry studies metric and geometric structures generated by differences, Difference Topology investigates connectivity, continuity, boundaries, neighborhoods, compactness, and invariant structures of Difference Space independently of specific distance values.

The theory establishes topological foundations for Difference Fields, Difference Networks, Difference Manifolds, and recursive Difference Systems, providing a framework for understanding how difference-generated structures preserve identity through transformation.

1. Introduction

Difference Geometry studies the shape of Difference Space.

Difference Topology studies the persistence of Difference Space.

The central question becomes:

Which properties of a system of differences remain unchanged when metric measurements are altered?

In Differentials, topology investigates structures generated by:

$\Delta \backslash \Delta \Delta$

rather than by points alone.

Therefore topology becomes a study of:

$T \Delta \backslash \mathcal{T}_{\Delta} T \Delta$

the topology induced by Difference Relations.

2. Topological Principle of Differentials

Axiom DT-1

Difference Topology arises from connectivity among differences.

Formally:

$T \Delta = \{U_i\} \backslash \mathcal{T}_{\Delta} = \{U_i\} T \Delta = \{U_i\}$

where:

$U_i \subseteq D, U_i \subseteq D$

represent open Difference Regions.

Interpretation

Difference Space is not primarily built from objects.

It is built from:

$$\Delta(A,B) \setminus \Delta(A,B) \Delta(A,B)$$

relations.

Topology emerges from how these relations connect.

3. Difference Neighborhoods

Definition

A Difference Neighborhood of a point A is the collection of differences directly connected to A .

$$N\Delta(A) = \{B \mid \Delta(A,B) \neq 0\} \quad N_{\Delta}(A) = \{B \mid \Delta(A,B) \neq 0\} \quad N\Delta(A) = \{B \mid \Delta(A,B) \neq 0\}$$

Interpretation

Neighborhoods define local environments of distinguishability.

They determine:

- local structure;
- local information flow;
- local transformation potential.

4. Open Difference Sets

Definition

A subset

$$U \subseteq DU \subseteq DU \subseteq D$$

is open if every element possesses a Difference Neighborhood entirely contained within the set.

$$\forall x \in U: \text{forall } x \in U: \forall x \in U: N_{\Delta}(x) \subseteq U \implies N_{\Delta}(x) \subseteq U$$

Meaning

Open Difference Sets represent regions where difference relations are internally preserved.

5. Closed Difference Sets

Definition

A set is closed if it contains all its Difference Limits.

$$U^- = \overline{U} \implies U^- = U$$

Interpretation

Closed Difference Sets correspond to stable Difference Structures.

They preserve identity under recursive transformations.

6. Difference Continuity

Definition

A Difference Transformation

$$f: D \rightarrow D \text{ if } f: D \rightarrow D$$

is continuous if it preserves Difference Neighborhoods.

$$f(N_{\Delta}(A)) \subseteq N_{\Delta}(f(A)) \implies f(N_{\Delta}(A)) \subseteq N_{\Delta}(f(A))$$

Interpretation

Continuity means:

No abrupt destruction of Difference Relations occurs.

Difference Identity remains preserved.

7. Difference Connectedness

Definition

Difference Space is connected if it cannot be separated into two disjoint nonempty open Difference Sets.

$$D = U \cup V \quad D = U \cup V \quad D = U \cup V$$

with

$$U \cap V = \emptyset \quad U \cap V = \emptyset \quad U \cap V = \emptyset$$

implies either:

$$U = \emptyset \quad U = \emptyset \quad U = \emptyset$$

or

$$V = \emptyset \quad V = \emptyset \quad V = \emptyset$$

Interpretation

Connected Difference Spaces represent unified systems.

Disconnected spaces represent independent Difference Domains.

8. Difference Boundaries

Definition

The Difference Boundary of a set U :

$$\partial U \text{ is the set of elements } x \text{ such that } x \in U \text{ and } x \notin U$$

contains all elements that separate internal and external Difference Regions.

Interpretation

Difference Boundaries correspond to transition zones.

They are locations where:

$$\Delta U \neq \emptyset$$

changes structural regime.

9. Difference Compactness

Definition

A Difference Space is compact if every open Difference Cover admits a finite subcover.

$$\forall \mathcal{U} \text{ (open cover of } U \text{)} \exists \mathcal{U}_f \subseteq \mathcal{U} \text{ (finite subcover of } U \text{)}$$

finite.

Interpretation

Compact Difference Systems possess finite structural descriptions despite potentially infinite local complexity.

10. Difference Holes

Definition

A Difference Hole is a region absent from the connectivity structure of Difference Space.

Formally:

$$H \subseteq DH \setminus \text{subseq} DH \subseteq D$$

such that connectivity paths around HHH exist but no path passes through HHH .

Interpretation

Difference Holes represent missing structural relations.

They often correspond to:

- forbidden transitions;
- missing information;
- inaccessible states.

11. Difference Homotopy

Definition

Two Difference Paths

$$\gamma_1 \setminus \gamma_1$$

and

$$\gamma_2 \setminus \gamma_2$$

are homotopic if one can be continuously transformed into the other.

$$\gamma_1 \simeq \gamma_2 \setminus \gamma_1 \setminus \gamma_2 \simeq \gamma_2$$

Interpretation

Homotopy classifies equivalent Difference Transformations.

Different trajectories may represent the same structural process.

12. Difference Invariants

Difference Topology seeks properties invariant under continuous deformation.

Examples:

Connectedness

$C \Delta C \setminus \Delta C$

Number of Difference Holes

$H \Delta H \setminus \Delta H$

Boundary Structure

$B \Delta B \setminus \Delta B$

Homotopy Classes

$\Pi \Delta \Pi \setminus \Delta \Pi$

These form the primary Difference Topological Invariants.

13. Difference Topological Identity Principle

Theorem DT-1

A Difference Structure preserves topological identity if all topological invariants remain unchanged.

Formally:

$$I\Delta(S_1)=I\Delta(S_2)I_{\Delta}(S_1)=I_{\Delta}(S_2)$$

where

$$I\Delta=(C\Delta,H\Delta,B\Delta,\Pi\Delta)I_{\Delta}=(C_{\Delta},H_{\Delta},B_{\Delta},\Pi_{\Delta})I\Delta=(C\Delta,H\Delta,B\Delta,\Pi\Delta)$$

Interpretation

Topological identity is deeper than geometric identity.

Metric properties may change.

Geometric properties may deform.

Topological identity persists.

14. Difference Fields as Topological Spaces

Every Difference Field

$$(D,\Delta)(D,\Delta)(D,\Delta)$$

induces a topology

$$(D,T\Delta)(D,\mathcal{T}_{\Delta})(D,T\Delta)$$

where Difference Relations define:

- neighborhoods;
- open sets;
- continuity;
- connectedness.

Thus Difference Fields become topological spaces.

15. Relation to Difference Geometry

Difference Geometry studies:

$g\Delta_{\Delta}$

(metric structure).

Difference Topology studies:

$T\Delta_{\Delta}$

(connectivity structure).

Geometry may change.

Topology remains.

Therefore:

$\text{Topology} \rightarrow \text{Geometry} \rightarrow \text{Metrics}$

forms a hierarchical organization of Difference Space.

16. Relation to Core Differentics Principles

Difference Topology provides formal foundations for:

Difference Relations $\rightarrow$ Structure

Topology defines global structure.

Difference Interactions $\rightarrow$ Dynamics

Continuity governs dynamic evolution.

Difference Recursion $\rightarrow$ Diversity

Topological branching generates diversity classes.

Difference CERELATH $\rightarrow$ Connectivity

Connectivity becomes the formal expression of causal linkage.

17. Conclusion

Difference Topology establishes the global invariant structure of Difference Space.

By introducing neighborhoods, continuity, connectedness, compactness, boundaries, holes, homotopy, and topological invariants, the theory extends Formal Differentics beyond metrics and geometry toward the study of persistent structural organization.

Difference Topology therefore serves as the mathematical framework for understanding how systems of differences maintain identity through transformation, deformation, recursion, and evolution.